Letters

Comments on “On the Computation of the Complete Spectral Green’s Dyadic for Layered Bianisotropic Structures”

E. L. Tan

In the above paper,[1] the authors have shown how to compute their complete spectral Green’s dyadic (CSGD) for multilayered bianisotropic planar structures. Their method consists of the following steps.

- Step 1) Transform normally directed current components into equivalent sheets of transverse currents.
- Step 2) Compute the transverse spectral Green’s dyadics and the transverse fields attributed to transverse currents.
- Step 3) Calculate the normal field components from the transverse fields.

While the authors have elaborated somewhat in steps 1 and 2, they have somehow treated step 3 in a rather *ad-hoc* manner by simply stating that, “the complete spectral (electric) Green’s dyadic can *finally* be obtained after algebraically calculating the normal component of the (electric) field from (its) transverse components.” Specifically, they refer the readers to [1] ([24] in the above paper) for an example of the *explicit* expressions for normally directed fields. As is well known, the dyadic Green’s function in its default definition represents the response due to point source of *arbitrary* orientation. Still, in view of the rationale behind [2] (not the 1971 edition in [8] of the above paper), as well as [3]–[5], it will be *incomplete* if one had not mentioned *explicitly* the source point/plane singularities for the Green’s function expansions in the space/spectral domain. These singularities cannot be deduced merely from the algebraic relations between normal E_z, H_z and transverse $\mathbf{E}_t, \mathbf{H}_t$.

In order to present the truly *complete* spectral Green’s dyadic, let us consider, for example, the expression of $G_{E_{z\mathrm{f}}J_{z\mathrm{s}}}$ defined and expanded as

$$E_{z\mathrm{f}}\left(\overline{k}_t, z\right) = \int \mathrm{d}z' G_{E_{z\mathrm{f}}J_{z\mathrm{s}}}\left(\overline{k}_t, z; z'\right) J_{z\mathrm{s}}\left(\overline{k}_t, z'\right)$$

$$\overline{k}_t = k_x\hat{x} + k_y\hat{y} \quad (1)$$

$$G_{E_{z\mathrm{f}}J_{z\mathrm{s}}} = G^{\delta}_{E_{z\mathrm{f}}J_{z\mathrm{s}}} + G^{[1]}_{E_{z\mathrm{f}}J_{z\mathrm{s}}}. \quad (2)$$

The indexes f and s in the subscripts correspond respectively to the field and source *layers* (not *interfaces* as in the equivalent boundary method). $G^{[1]}_{E_{z\mathrm{f}}J_{z\mathrm{s}}}$ denotes the Green’s dyadic element in the above paper, which, as asserted by the authors, can be obtained from the algebraic relations between normal and transverse fields. For complete representation in the spectral domain, one must specify as well the source plane term $G^{\delta}_{E_{z\mathrm{f}}J_{z\mathrm{s}}}$, which can be determined to be

$$G^{\delta}_{E_{z\mathrm{f}}J_{z\mathrm{s}}} = -\frac{\mu_{zz\mathrm{s}}}{j\omega\Delta_{\mathrm{s}}}\delta\left(z - z'\right)\delta_{\mathrm{fs}}. \quad (3)$$

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[1] F. L. Mesa, R. Marqués, and M. Horno, *IEEE Trans. Microwave Theory Tech.*, vol. 46, no. 8, pp. 1158–1164, Aug. 1998.

$\delta(z - z')$ is the Dirac delta function, $\Delta_{\mathrm{s}} = \epsilon_{zz\mathrm{s}}\mu_{zz\mathrm{s}} - \xi_{zz\mathrm{s}}\zeta_{zz\mathrm{s}}$, and the (absolute) constitutive dyadics are $\overline{\overline{\epsilon}} = \epsilon_0\overline{\epsilon}_r, \overline{\overline{\mu}} = \mu_0\overline{\mu}_r, \overline{\overline{\xi}} = (1/c)\overline{\xi}_r, \overline{\overline{\zeta}} = (1/c)\overline{\eta}_r$. Note that we have included the Kronecker delta symbol δ_{fs} to demand the unambiguous specification of the source layer, especially for normal sources located *on the interface* between two different media. Apparently, this term contains some form similar to that of (8) in the above paper, but has not been associated by them to the final source plane singularity of the CSGD. Perhaps such association is slightly more noticeable in [6, eq. (40)] ([21] in the above paper) or from the *source-incorporated* normal field expressions in [6, eqs. (27), (28)], rather than from the *source-free* expressions of [1] referred by the authors. Indeed, by conforming to the coordinate system in Fig. 1 in the above paper, we have the source-incorporated normal fields in bianisotropic layers given by, e.g.,

$$\begin{aligned} E_{z\mathrm{f}} = &-\left[\frac{\xi_{zz\mathrm{f}}}{\Delta_{\mathrm{f}}}(\overline{\kappa}_{zt} - \overline{\zeta}_{zt\mathrm{f}}) + \frac{\mu_{zz\mathrm{f}}}{\Delta_{\mathrm{f}}}\overline{\epsilon}_{zt\mathrm{f}}\right]\cdot\mathbf{E}_{t\mathrm{f}} \\ &+\left[\frac{\xi_{zz\mathrm{f}}}{\Delta_{\mathrm{f}}}\overline{\mu}_{zt\mathrm{f}} - \frac{\mu_{zz\mathrm{f}}}{\Delta_{\mathrm{f}}}(\overline{\kappa}_{zt} + \overline{\xi}_{zt\mathrm{f}})\right]\cdot\mathbf{H}_{t\mathrm{f}} \\ &-\frac{1}{j\omega\Delta_{\mathrm{s}}}\Big[\mu_{zz\mathrm{s}}J_{z\mathrm{s}} - \xi_{zz\mathrm{s}}M_{z\mathrm{s}}\Big]\delta_{\mathrm{fs}} \end{aligned} \quad (4)$$

$\overline{\kappa}_{zt} = (-k_y\hat{x} + k_x\hat{y})/\omega, \overline{\epsilon}_{zt\mathrm{f}} = \epsilon_{zx\mathrm{f}}\hat{x} + \epsilon_{zy\mathrm{f}}\hat{y}$, etc. Notice that there is no Dirac delta function adjunct to the source terms like (20) in the above paper or [6, eqs. (27), (28)]. This is because it would be more instructive to associate such functions with Green’s dyadics (rather than field/source vectors) for they are the ones to be treated (integrated) in the sense of distributions.

Moving on to the numerical examples in the above paper, it is found that some plots presented by the authors are erroneous and cannot be reproduced using the data specified. Consider first the comparison of Fig. 4 in the above paper with the results from [7] ([25] in the above paper) and [8] ([18] in the above paper). For the parameters given therein (cf [7, Fig. 7] and [8, Fig. 12(b)]), one will observe the discrepancies in the plots for $\phi_H = 2\pi/3$ in Fig. 4 in the above paper. The correct $|E_\theta|_r$ and $|E_\phi|_r$ for this case, as well as for $\phi_H = \pi/3$, are presented in Fig. 1, with the amplitudes normalized to $|E_\phi|_{\max}$ when $\phi_H = 0$. Apparently, the plots for $\phi_H = 2\pi/3$ in Fig. 4 in the above paper should be attributed to $\phi_H = \pi/3$, which may appear to be of typographical error in [8], but would be misleading if reasserted by the above paper.

In Fig. 5 in the above paper, it is indeed confusing to especially observe that the plot for $\theta_{\mathrm{dip}} = 0^\circ$ does not pass through the sampled data from the case $\hat{p} = \hat{z}$ of [9, Fig. 2] and [10] ([13] in the above paper), yet the authors have claimed their data “fits properly” without further explanation in the discussion. Since the form of permittivity dyadic has not been supplied explicitly by the above paper, and to be specific, we shall write down its elements for the tilted uniaxial substrate by referring to [9] and [10] (with $\phi_{\mathrm{ax}} = 0^\circ$ and varying θ_{ax})

$$[\overline{\overline{\epsilon}}_1]_{xyz} = \begin{bmatrix} \epsilon_u\cos^2\theta_{\mathrm{ax}} + \epsilon_v\sin^2\theta_{\mathrm{ax}} & 0 & (\epsilon_v - \epsilon_u)\cos\theta_{\mathrm{ax}}\sin\theta_{\mathrm{ax}} \\ 0 & \epsilon_u & 0 \\ (\epsilon_v - \epsilon_u)\cos\theta_{\mathrm{ax}}\sin\theta_{\mathrm{ax}} & 0 & \epsilon_u\sin^2\theta_{\mathrm{ax}} + \epsilon_v\cos^2\theta_{\mathrm{ax}} \end{bmatrix}. \quad (5)$$

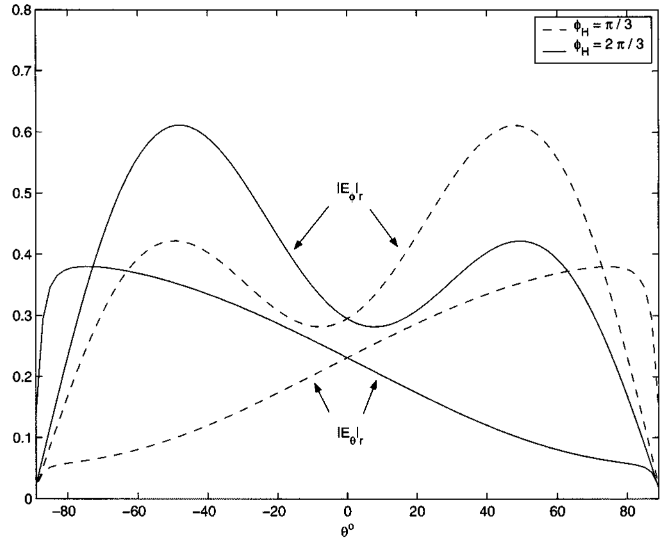

Fig. 1. $|E_\theta|_r$ and $|E_\phi|_r$ relative far-field amplitudes (normalized to $|E_\phi|_{\max}, \phi_H = 0$) in the $\phi = \pi/2$ plane for $\phi_H = \pi/3, 2\pi/3$. Other parameters such as dipole orientation, layer structure, and materials are the same as Fig. 4 in the above paper.

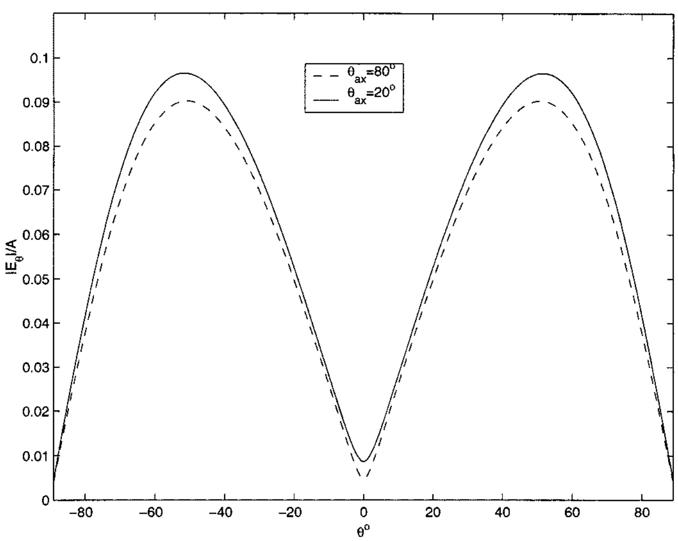

Fig. 2. $|E_\theta|/A$ relative far-field amplitude (with $A = |E_\theta|_{\max}, \theta_{\rm dip} = 90^\circ$, $\theta_{\rm ax} = 90^\circ$) in the $\phi = 0$ plane for a vertical electric dipole located *just below* (inside) the top of a grounded tilted uniaxial substrate. Other parameters such as layer structure and materials are the same as in Fig. 5 in the above paper.

Here, the layer index 1 refers to the substrate, ϵ_u and ϵ_v are defined for $\theta_{\rm ax} = 0^\circ$, i.e., $\epsilon_u = \epsilon_{xx1}|_{\theta_{\rm ax}=0^\circ} = \epsilon_{yy1}|_{\theta_{\rm ax}=0^\circ}$, and $\epsilon_v = \epsilon_{zz1}|_{\theta_{\rm ax}=0^\circ}$. To resolve the dilemma of "proper fit" (whether in the above paper or [9] and [10] is correct), the parameters in (5) are set according to the above paper, i.e., $\theta_{\rm ax} = 20^\circ, \epsilon_u = 10.7\epsilon_0, \epsilon_v = 10.4\epsilon_0$, and the results are recomputed for vertical dipole in Fig. 2. The normalization, which has not been specified clearly in the above paper, is defined here to be $A = |E_\theta|_{\max}$ when $\theta_{\rm dip} = 90^\circ, \theta_{\rm ax} = 90^\circ$. Moreover, it is necessary to specify unambiguously the location of vertical dipole *on the interface*, i.e., whether it is just above or below the interface. This can be justified from the large difference in the scale of

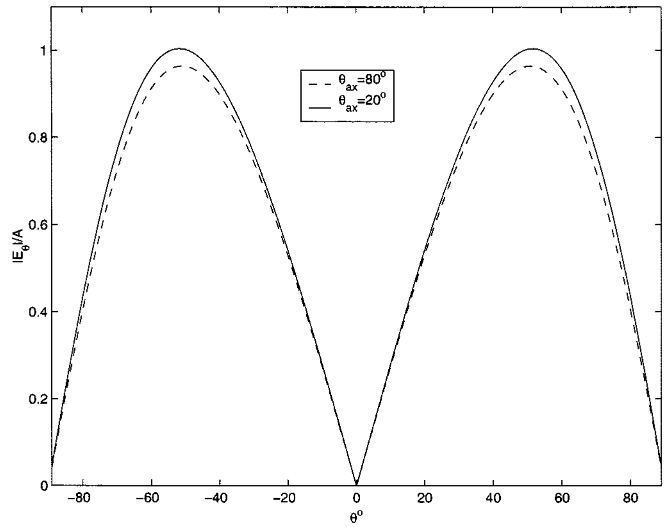

Fig. 3. Same as Fig. 2, except that the vertical dipole is located *just above* the top of substrate (in air).

Figs. 2 and 3, and by noting the variation around $\theta = 0^\circ$ (masked off by the symbols in Fig. 5 in the above paper). For more comparison, we include as well the case for $\theta_{\rm ax} = 80^\circ$ and revisit the results of [9] and [10], which are confirmed to be incorrect only recently [cf [11, p. 85] (yet still "fitted properly" in the above paper)].

Without pursuing further other numerical examples, it is noted that there are still other doubts contained in the above paper, which deserve clarification from the authors, particularly the following:

1) inconsistent use of the coordinate systems in the above paper and [12] (cited frequently by the authors as [15] in the above paper), thus causing much confusion in the application of formulas from [12], definition of *transverse* $\mathbf{k}_t = k_x\hat{x} + k_z\hat{z}$, and specification of constitutive parameters (e.g., $\epsilon_{xx} = \epsilon_{zz}$ or $\epsilon_{xx} = \epsilon_{yy}$?);
2) incomplete specification of the source layers for their normal sources *on the interface* , e.g., the title of Section III in the above paper indicates dipole embedding *in* complex layered media, but their reference of [9] and [10] (with "fit" comparison) assumes dipole in free space;
3) incomplete definition of the *impedance* matrix $[Z]$, noting the formulas for the *admittance* matrices $[\mathbf{g}^-]_{0,0}$ and $[\mathbf{g}^+]_{N,N}$ in (39) and (40) in the above paper;
4) incomplete treatment of other types of boundary wall conditions, e.g., singular $[Y]$ (dual of singular $[Z]$ in 3), despite the claim of applicability for any kind of impedance/admittance dyadics.

Finally, the above paper is incomplete by not citing the relevant work of [13].

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Authors' Reply

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We very much appreciate Prof. Tan's comments regarding the above paper.[1] As suggested by Prof. Tan, the complete expression for the E_z field at the source plane requires the introduction of the corresponding singularity terms. This is obvious and we should recognize that, in the last paragraph of Section II-B in the above paper, we were implicitly excluding the source plane. Certainly we were aware of these singularities since their presence was the basis of our treatment—see (8) and (9) in the above paper. We are also thankful that Prof. Tan's comments pointed out a typographical error in Fig. 4 of the above paper.

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